

Single-photon-assisted entanglement concentration of a multi-photon system in a partially entangled W state with weak cross-Kerr nonlinearity*

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We propose a nonlocal entanglement concentration protocol (ECP) for N -photon systems in a partially entangled W state, resorting to some ancillary single photons and the parity-check measurement based on cross-Kerr nonlinearity. One party in quantum communication first performs a parity-check measurement on her photon in an N -photon system and an ancillary photon, and then she picks up the even-parity instance for obtaining the standard W state. When she obtains an odd-parity instance, the system is in a less-entanglement state and it is the resource in the next round of entanglement concentration. By iterating the entanglement concentration process several times, the present ECP has the total success probability approaching to the limit in theory. The present ECP has the advantage of a high success probability. Moreover, the present ECP requires only the N -photon system itself and some ancillary single photons, not two copies of the systems, which decreases the difficulty of its implementation largely in experiment. It maybe have good applications in quantum communication in future.

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I. INTRODUCTION

Entanglement is a key important resource in quantum information and quantum computation [1]. The advantage of quantum computer, the powerful computation, comes from multipartite entanglement, compared with classical computer. Also, entanglement is used as the information carries in quantum communication, such as quantum key distribution (QKD) [2–6], quantum teleportation [7], quantum dense coding [8, 9], quantum secret sharing [10–17], quantum state sharing [18–22], controlled teleportation [23–25], and so on. In a long-distance quantum communication, entanglement is used to construct quantum repeaters. However, entanglement is fragile to channel noise. In a practical transmission or the process for storing an entangled quantum system, it inevitably suffers from channel noise and its environment. The noise will make the system decoherent, which will decrease the security of QKD protocols and the fidelity of quantum teleportation and dense coding. There are some interesting ways for dealing with the issue of decoherence in quantum communication, such as decoherence-free subspaces [26–29], faithful qubit distribution [30, 31], faithful qubit transmission [32], error-rejecting codes [33], faithful entanglement distribution [34], and so on. Most of them are very useful for overcoming a collective noise by encoding a logical qubit with several physical qubits. There is a fundamental hypothesis that the noise is a collective one. These methods are used to deal with the photon systems before they are transmitted over a noisy channel.

Entanglement purification and entanglement concentration are two interesting quantum techniques with which the users can obtain some high-fidelity entangled photon systems after they are transmitted over a noisy channel or stored in a practical environment and they are in a less-entanglement state. In

detail, entanglement purification is used to extract some high-fidelity entangled systems from a less-entangled ensemble in a mixed state [35–46]. Entanglement concentration is used to obtain a subset of photon systems in a maximally entangled state from a set of systems in a partially entangled pure state. Although entanglement purification is more general than entanglement concentration in the practical applications because an entangled photon system is usually in a mixed entangled state after it is transmitted over a noisy channel, entanglement concentration is more efficient for the two remote parties in quantum communication, say the sender Alice and the receiver Bob, to distill some maximally entangled systems from an ensemble in a less-entangled pure state because entanglement purification should consume a great deal of quantum resource for improving the fidelity of systems in a mixed entangled state, not obtain a maximally entangled state directly. Entanglement concentration is useful in some particular cases, such as the decoherence of entanglement arising from the storage process or the imperfect entanglement source.

Since Bennett *et al.* [47] proposed the original entanglement concentration protocol (ECP) in 1996, there have been some interesting and typical ECPs for photon systems [47–55] and atom systems [56, 57]. For example, Bose *et al.* [48] proposed an ECP based on entanglement swapping in 1999. Subsequently, Shi *et al.* [49] presented an ECP based on a collection unitary evolution on a qubit in a multi-qubit system and an ancillary qubit. In 2001, an ECP based on polarizing beam splitters (PBSs) was proposed [50, 51]. In 2008, Sheng, Deng and Zhou proposed an interesting ECP [52] with cross-Kerr nonlinearities. In 2010, they presented the first single-photon ECP [53] with cross-Kerr nonlinearities. In 2012, Sheng *et al.* [54] proposed a single-photon-assisted ECP for partially entangled multi-photon systems. Recently, an optimal nonlocal multipartite ECP for photon systems in a partially entangled Bell-type state is proposed [55], resorting to a parity-check measurement on one photon in the system and an ancillary single photon and the projection measurement on the ancillary photon with cross-Kerr nonlinearities.

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Although there exist some interesting ECPs, most of them are used to distill some maximally entangled Bell states or Greenberger-Horne-Zeilinger (GHZ) states. There are few schemes for concentrating the non-maximally entangled pure W-class states. In essence, W state are inequivalent to the GHZ states as they cannot be converted into each other under stochastic local operations and classical communication (SLOCC). Moreover, a W state is more robust than a GHZ state against the loss of one or two photons. Therefore, it is interesting to discuss the entanglement concentration on the partially entangled W state. By far, there are three ECPs for photon systems in a partially entangled W state [58–60]. The first one is a linear optical scheme for entanglement concentration of two known partially entangled three-photon W states [58]. The second one is linear-optics-based entanglement concentration of unknown partially entangled three-photon W states [59]. It is proposed by Wang, Zhang, and Yeon [59] in 2010. In 2011, Xiong and Ye [60] proposed another ECP for a partially entangled W state with cross-Kerr nonlinearity. Both these two interesting ECPs are used to deal with an unknown multi-photon W-class state. There is no ECP for a known multi-photon W-class state.

In this paper, we proposed an nonlocal ECP for N -photon systems in a known partially entangled pure W state, resorting to ancillary single photons and the parity-check measurement based on cross-Kerr nonlinearity. It does not depend on two copies of N -photon systems in a partially entangled W-class state in each round of concentration, just each system itself and some ancillary single photons, which makes it far different from other ECP for W states [59, 60]. In the present ECP, only one of the parties in quantum communication, say Alice, first operates her photon and the ancillary single photons for concentrating the entanglement of an N -photon system and then tells the others to retain or discard the system, which greatly simplifies the complication of classical communication as others require all the parties operate the entanglement process in the same way, similar to the works for a Bell-type state [54, 55]. Moreover, the present ECP has a higher total success probability approaching to the limit in theory by iterating the entanglement concentration process several times. All these advantages make our ECP more feasible than others. It maybe have good applications in quantum communication in future.

II. ENTANGLEMENT CONCENTRATION OF PARTIALLY ENTANGLED THREE-PHOTON W STATES

Before we discuss the principle of our ECP for a partially entangled three-photon W states, we first introduce the principle of a parity-check detector (PCD) on the polarization states of two photons with cross-Kerr nonlinearity. In fact, the principle of the PCD here is similar to those in Refs.[46, 55, 61]. In detail, the Hamiltonian of a cross-Kerr nonlinearity can be written as follows [61]:

$$H_{ck} = \hbar\chi a_s^\dagger a_s a_p^\dagger a_p \quad (1)$$

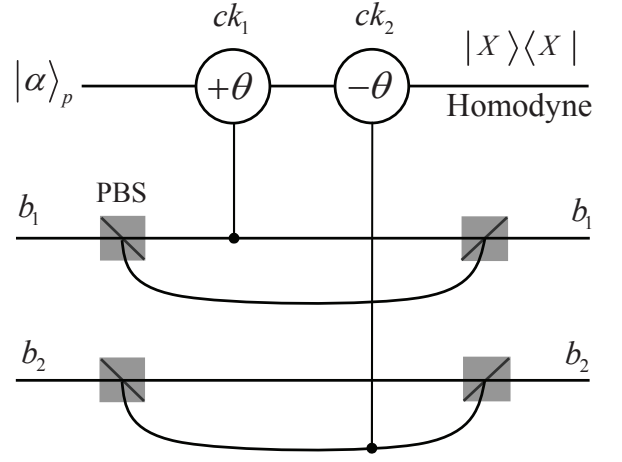


FIG. 1: The principle of a parity-check detector (PCD) on the polarizations of two photons, the same as that in Ref.[46, 55]. PBS represents a polarizing beam splitter which transmits the photon in the horizontal polarization $|H\rangle$ and reflects the photon in the vertical polarization $|V\rangle$. ck_1 and ck_2 represent two cross-Kerr nonlinearities which will lead to the phase shift $+\theta$ and $-\theta$ when there is a photon passing through the media, respectively. $|X\rangle\langle X|$ represents an X quadrature measurement with which one can not distinguish the states $|\alpha e^{\pm i\theta}\rangle_p$ [61, 62].

where a_s^\dagger and a_p^\dagger are the creation operations, and a_s and a_p are the destruction operations. The subscripts s and p represent the signal light and the probe light, respectively. χ is the coupling strength of the cross-Kerr nonlinearity. If a signal light in the state $|\Psi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$ ($|0\rangle_s$ and $|1\rangle_s$ denote that there are no photon and one photon respectively in this state) and a coherent probe beam in the state $|\alpha\rangle_p$ couple with a cross-Kerr nonlinearity medium, the evolution of the whole system can be described as [46, 55, 61]:

$$\begin{aligned} U_{ck}|\Psi\rangle_s|\alpha\rangle_p &= e^{iH_{ck}t/\hbar}[c_0|0\rangle_s + c_1|1\rangle_s]|\alpha\rangle_p \\ &= c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p, \end{aligned} \quad (2)$$

where $\theta = \chi t$ is the phase shift of the probe beam, which depends on the interaction time t and the coupling strength χ . That is, the coherent beam P picks up a phase shift θ directly proportional to the number of the photons in the signal light in the Fock state $|\Psi\rangle_s$. Based on this feature of a cross-Kerr nonlinearity, the principle of our PCD is shown in Fig.1, similar to those in Refs. [46, 55, 61]. Here $|X\rangle\langle X|$ represents an X quadrature measurement with which one can not distinguish the states $|\alpha e^{\pm i\theta}\rangle_p$ [61, 62]. With the two cross-Kerr nonlinearities ck_1 and ck_2 , one can distinguish the superpositions and mixtures of the polarization states $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ based on the different phase shifts. That is, the probe beam P will pick up a phase shift θ if the two photons is in the state $|HH\rangle_{b_1b_2}$ or $|VV\rangle_{b_1b_2}$, while it picks up a phase shift 0 when the two photons is in the state $|VH\rangle_{b_1b_2}$ or $|HV\rangle_{b_1b_2}$. In other words, when the parity of the two photons is even, the coherent beam will pick up a phase shift θ ; otherwise it will pick up a phase shift 0. By detecting the phase

shift of the probe beam, one can determine that the parity of the two photons is even or odd.

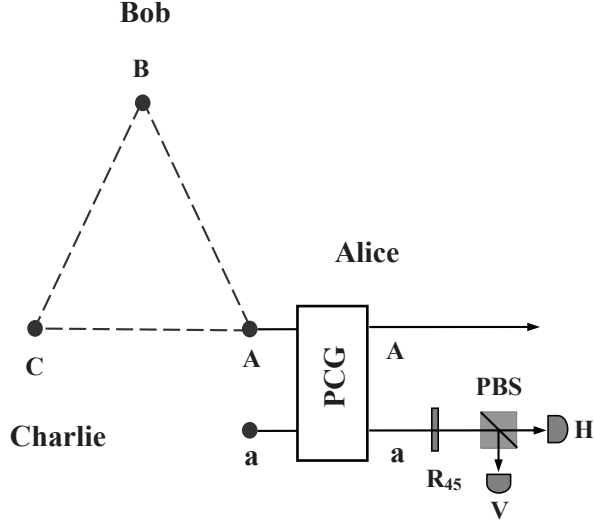


FIG. 2: The principle of the present ECP for three-photon W states with ancillary single photons and the parity-check detector (PCD) based on cross-Kerr nonlinearity. R_{45} represents a Hadamard operation on the polarization of the ancillary single photon a . H and V represent the horizontal polarization state of photons $|H\rangle$ and the vertical polarization state $|V\rangle$, respectively.

Let us assume that the three-photon system composed of the three photons ABC is in the following partially entangled W-class polarization states:

$$|\varphi\rangle_{CBA} = \alpha|H\rangle_C|H\rangle_B|V\rangle_A + \beta(|H\rangle_C|V\rangle_B|H\rangle_A + |V\rangle_C|H\rangle_B|H\rangle_A), \quad (3)$$

where the subscripts A , B , and C represent the three photons belonging to the three remote parties in quantum communication, say Alice, Bob, and Charlie. Different from those ECPs for W states [59, 60], here α and β are two known real numbers and satisfy the relation

$$\alpha^2 + 2\beta^2 = 1. \quad (4)$$

Certainly, in a practical application, it is not difficult for the parties to obtain information about the parameters α and β by detecting a subset of three-photon systems, similar to the case for Bell-type states [55].

The principle of our ECP is shown in Fig.2. In the process of concentrating a three-photon system, Alice prepares an ancillary photon a . It is in the polarization state $|\varphi\rangle_a$. Here

$$|\varphi\rangle_a = \frac{1}{\sqrt{\alpha^2 + \beta^2}}(\alpha|H\rangle + \beta|V\rangle)_a. \quad (5)$$

Before Alice performs a parity-check measurement on her photon A and the ancillary photon a , the composite system composed of the four photons $CBAa$ is in the state

$$|\Phi\rangle_{CBAa} = |\varphi\rangle_{CBA} \otimes |\varphi\rangle_a$$

$$\begin{aligned} &= \frac{1}{\sqrt{\alpha^2 + \beta^2}} \{ \alpha\beta[|H\rangle_C|H\rangle_B|V\rangle_A|V\rangle_a + (|H\rangle_C|V\rangle_B \\ &+ |V\rangle_C|H\rangle_B)|H\rangle_A|H\rangle_a] + \alpha^2|H\rangle_C|H\rangle_B|V\rangle_A|H\rangle_a \\ &+ \beta^2(|H\rangle_C|V\rangle_B + |V\rangle_C|H\rangle_B)|H\rangle_A|V\rangle_a \}. \end{aligned} \quad (6)$$

With the parity-check measurement on the photons A and a , Alice can divide the state of the four-photon system $CBAa$ into two classes. In the first one, it is in a state in which each item has the same parameter, that is,

$$\begin{aligned} |\Psi_1\rangle_{CBAa} &= \frac{1}{\sqrt{3}}(|H\rangle_C|H\rangle_B|V\rangle_A|V\rangle_a + (|H\rangle_C|V\rangle_B \\ &+ |V\rangle_C|H\rangle_B)|H\rangle_A|H\rangle_a). \end{aligned} \quad (7)$$

In the second one, the system is in a state with less entanglement and different parameters, that is,

$$\begin{aligned} |\Psi'_1\rangle_{CBAa} &= \frac{1}{\sqrt{\alpha^2 + 2\beta^2}} [\alpha^2|H\rangle_C|H\rangle_B|V\rangle_A|H\rangle_a \\ &+ \beta^2(|H\rangle_C|V\rangle_B + |V\rangle_C|H\rangle_B)|H\rangle_A|V\rangle_a]. \end{aligned} \quad (8)$$

In the fact, in the first class, Alice obtains an even parity when she performs a parity-check measurement on the photon A and the ancillary a . The state $|\Psi_1\rangle_{CBAa}$ corresponds to the parameter $\alpha\beta$ in Eq.(6). In the second class, Alice obtains an odd parity, which leads to the state $|\Psi'_1\rangle_{CBAa}$. The probability that Alice obtains an even parity when she measures the two photons A and a is

$$P_1 = \frac{3\alpha^2\beta^2}{\alpha^2 + \beta^2}. \quad (9)$$

The probability that Alice obtains an odd parity is

$$P'_1 = \frac{\alpha^4 + 2\beta^4}{\alpha^2 + \beta^2}. \quad (10)$$

Alice can measure the ancillary photon a for obtaining the standard three-photon W state from the four-photon state $|\Psi_1\rangle_{CBAa}$ with the basis X (i.e., $\{|\pm x\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)\}$). When she obtain the state $|+x\rangle_a$, the three-photon system is in the standard W state $|W_3^+\rangle$. Here

$$|W_3^+\rangle_{CBA} = \frac{1}{\sqrt{3}}(|H\rangle_C|H\rangle_B|V\rangle_A + (|H\rangle_C|V\rangle_B + |V\rangle_C|H\rangle_B)|H\rangle_A). \quad (11)$$

When she obtain the state $|-x\rangle_a$, the three-photon system is in another standard W stat

$$|W_3^-\rangle_{CBA} = \frac{1}{\sqrt{3}}(|H\rangle_C|H\rangle_B|V\rangle_A - (|H\rangle_C|V\rangle_B + |V\rangle_C|H\rangle_B)|H\rangle_A). \quad (12)$$

Alice can transform the state $|W_3^-\rangle_a$ into the state $|W_3^+\rangle$ by performing a phase-flip operation $\sigma_z = |H\rangle\langle H| - |V\rangle\langle V|$ on her photon A .

As for the less-entanglement state $|\Psi'_1\rangle_{CBAa}$, Alice can measure the ancillary photon a with the basis X to transform it into a three-photon state with less entanglement. That is,

$$|\Psi'_2\rangle_{CBA} = \frac{\alpha^2}{\sqrt{\alpha^2 + 2\beta^2}} |H\rangle_C |H\rangle_B |V\rangle_A + \frac{\beta^2}{\sqrt{\alpha^2 + 2\beta^2}} (|H\rangle_C |V\rangle_B + |V\rangle_C |H\rangle_B) |H\rangle_A. \quad (13)$$

In detail, when Alice obtains the state $|+x\rangle_a$, the three-photon system is in the state $|\Psi'_2\rangle_{CBA}$. When Alice obtains the state $|-x\rangle_a$, she need only perform a phase-flip operation on her photon A and she will obtain the state $|\Psi'_2\rangle_{CBA}$.

It is not difficult to find that the state $|\Psi'_2\rangle_{CBA}$ shown in Eq.(13) has the same form as the state $|\varphi\rangle_{CBA}$ shown in Eq.(3) but different parameters. We need only replace the parameters α and β in Eq.(3) with the parameters $\alpha' \equiv \frac{\alpha^2}{\sqrt{\alpha^2 + 2\beta^2}}$ and $\beta' \equiv \frac{\beta^2}{\sqrt{\alpha^2 + 2\beta^2}}$, respectively. That is, Alice can also concentrate the state $|\Psi'_2\rangle_{CBA}$ as the same as the state $|\varphi\rangle_{CBA}$. The probability that Alice, Bob, and Charlie obtain the standard three-photon W state from each system in the state $|\Psi'_2\rangle_{CBA}$ is

$$P_2 = \frac{3 \cdot \frac{\alpha^4}{\alpha^4 + 2\beta^4} \cdot \frac{\beta^4}{\alpha^4 + 2\beta^4}}{\frac{\alpha^4}{\alpha^4 + 2\beta^4} + \frac{\beta^4}{\alpha^4 + 2\beta^4}} = \frac{3\alpha^4\beta^4}{(\alpha^4 + \beta^4)(\alpha^4 + 2\beta^4)}. \quad (14)$$

Certainly, the probability that Alice, Bob, and Charlie obtain the three-photon state with less entanglement from each system in the state $|\Psi'_2\rangle_{CBA}$ becomes

$$P'_2 = \frac{(\frac{\alpha^4}{\alpha^4 + 2\beta^4})^2 + 2 \cdot (\frac{\beta^4}{\alpha^4 + 2\beta^4})^2}{\frac{\alpha^4}{\alpha^4 + 2\beta^4} + \frac{\beta^4}{\alpha^4 + 2\beta^4}} = \frac{\alpha^8 + 2\beta^8}{(\alpha^4 + \beta^4)(\alpha^4 + 2\beta^4)}. \quad (15)$$

After Alice performs the entanglement concentration process for n times, the total probability that Alice, Bob, and Charlie obtain the standard three-photon W state $|W_3^+\rangle_{CBA}$ is

$$\begin{aligned} P(n) &= P_1 + P'_1 P_2 + P'_1 P'_2 P_3 + \dots + P'_1 P'_2 \dots P'_{n-1} P_n \\ &= 3 \left[\frac{\alpha^2 \beta^2}{\alpha^2 + \beta^2} + \frac{\alpha^4 \beta^4}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)} \right. \\ &\quad + \frac{\alpha^8 \beta^8}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)(\alpha^8 + \beta^8)} + \dots \\ &\quad \left. + \frac{\alpha^{2^n} \beta^{2^n}}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4) \dots (\alpha^{2^n} + \beta^{2^n})} \right]. \end{aligned} \quad (16)$$

Let us assume that the parameter $|\alpha|^2 \leq |\beta|^2$. One can see that the maximal success probability that Alice, Bob, and Charlie can distill a standard W state from the partially entangled state $|\varphi\rangle_{CBA} = \alpha|H\rangle_C |H\rangle_B |V\rangle_A + \beta(|H\rangle_C |V\rangle_B |H\rangle_A + |V\rangle_C |H\rangle_B |H\rangle_A)$ is $3|\alpha|^2$ and $|\alpha|^2 \in [0, 1/3]$. Let us assume $F = 3|\alpha|^2$. The relation between the total probability $P(n)$ and F is shown in Fig.3. Generally, when Alice repeats her entanglement concentration 5 times, the total success probability $P(n)$ approaches to the parameter F , the limit in theory.

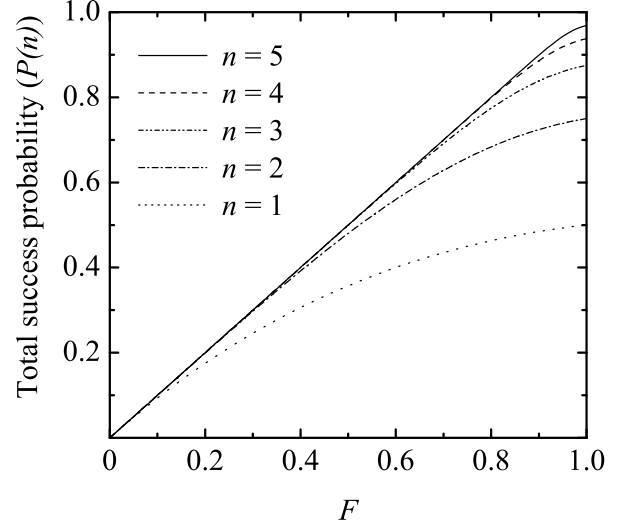


FIG. 3: The relation between the total success probability $P(n)$ and the parameter $F = 3|\alpha|^2$ when $|\alpha|^2 \leq |\beta|^2$ for the cases $n = 1$ (dot curve), 2 (dash-dot curve), 3 (dash-dot-dot curve), 4 (dash curve), and 5 (solid curve), respectively.

For a partially entangled W-class state with less entanglement, Alice need only iterate the process for 2 or 3 times for obtaining the total success probability approaching to the limit.

III. ENTANGLEMENT CONCENTRATION OF PARTIALLY ENTANGLED N -PHOTON W STATES

In principle, it is not difficult to generalize our ECP for partially entangled N -photon W states. Let us assume that there is a partially entangled N -photon W-class state

$$\begin{aligned} |\varphi\rangle_{ABC\dots Z} &= \alpha_1 |H\rangle_Z \dots |H\rangle_C |H\rangle_B |V\rangle_A \\ &\quad + \beta_1 (|H\rangle_Z \dots |H\rangle_C |V\rangle_B |H\rangle_A \\ &\quad + |H\rangle_Z \dots |V\rangle_C |H\rangle_B |H\rangle_A + \dots \\ &\quad + |V\rangle_Z \dots |H\rangle_C |H\rangle_B |H\rangle_A). \end{aligned} \quad (17)$$

The subscript A, B, C, \dots , and Z represent the photons in W-class states shared by Alice, Bob, Charlie, \dots , and Zach, respectively. Here, the parameters α_1 and β_1 satisfy the following relation

$$\alpha_1^2 + (N-1)\beta_1^2 = 1. \quad (18)$$

For obtain a standard N -photon W state from each system in the state $|\varphi\rangle_{ABC\dots Z}$, Alice prepares an ancillary photon a_1 in the state $|\varphi\rangle_{a_1} = \frac{1}{\sqrt{\alpha_1^2 + \beta_1^2}} (\alpha_1 |H\rangle + \beta_1 |V\rangle)_{a_1}$, similar to the case in the entanglement concentration of a three-photon system. Then the state of the composite system can be written as

$$\begin{aligned} |\Phi\rangle_{Z\dots CBAa_1} &= |\varphi\rangle_{Z\dots CBA} \otimes |\varphi\rangle_{a_1} \\ &= \frac{1}{\sqrt{\alpha_1^2 + \beta_1^2}} \{ \alpha_1 \beta_1 [|H\rangle_Z \dots |H\rangle_C |H\rangle_B |V\rangle_A |V\rangle_{a_1} \end{aligned}$$

$$\begin{aligned}
& + (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
& + \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A |H\rangle_{a_1} \\
& + \alpha_1^2 |H\rangle_Z \cdots |H\rangle_C |H\rangle_B |V\rangle_A |H\rangle_{a_1} \\
& + \beta_1^2 (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
& + \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A |V\rangle_{a_1} \}. \quad (19)
\end{aligned}$$

If the parity of the two photons A and a is even, the $(N+1)$ -photon system is in the state

$$\begin{aligned}
|\Psi''_1\rangle_{Z \cdots CBAa_1} &= \frac{1}{\sqrt{N}} [|H\rangle_Z \cdots |H\rangle_C |H\rangle_B |V\rangle_A |V\rangle_{a_1} \\
&+ (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
&+ \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A |H\rangle_{a_1}], \quad (20)
\end{aligned}$$

which takes place with the probability

$$P''_1 = \frac{N\alpha_1^2\beta_1^2}{\alpha_1^2 + \beta_1^2}. \quad (21)$$

If the parity of the two photons A and a_1 is odd, the $(N+1)$ -photon system is in the state

$$\begin{aligned}
|\Psi'''_1\rangle_{Z \cdots CBAa_1} &= \frac{1}{\sqrt{\alpha_1^4 + (N-1)\beta_1^4}} [\alpha_1^2 |H\rangle_Z \cdots |H\rangle_C |H\rangle_B |V\rangle_A |H\rangle_{a_1} \\
&+ \beta_1^2 (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
&+ \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A |V\rangle_{a_1}], \quad (22)
\end{aligned}$$

which takes place with the probability

$$P'''_1 = \frac{\alpha_1^4 + (N-1)\beta_1^4}{\alpha_1^2 + \beta_1^2}. \quad (23)$$

By measuring the ancillary photon a_1 in the $(N+1)$ -photon system in the state $|\Psi_1\rangle_{Z \cdots CBAa_1}$, the N parties can obtain the standard N -photon state

$$\begin{aligned}
|W_N^+\rangle_{Z \cdots CBA} &= \frac{1}{\sqrt{N}} [|H\rangle_Z \cdots |H\rangle_C |H\rangle_B |V\rangle_A \\
&+ (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
&+ \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A] \quad (24)
\end{aligned}$$

with or without a phase-flip operation on the photon A . When the $(N+1)$ -photon system is in the state $|\Psi'''_1\rangle_{Z \cdots CBAa_1}$, Alice can collapse it into the N -photon state

$$\begin{aligned}
|\Psi'_1\rangle_{Z \cdots CBA} &= \frac{1}{\sqrt{\alpha_1^4 + (N-1)\beta_1^4}} [\alpha_1^2 |H\rangle_Z \cdots |H\rangle_C |H\rangle_B |V\rangle_A \\
&+ \beta_1^2 (|H\rangle_Z \cdots |H\rangle_C |V\rangle_B + |H\rangle_Z \cdots |V\rangle_C |H\rangle_B \\
&+ \cdots + |V\rangle_Z \cdots |H\rangle_C |H\rangle_B) |H\rangle_A] \quad (25)
\end{aligned}$$

by measuring the ancillary photon a_1 with the basis X and performing a phase-flip operation or not on the photon A . Moreover, the state $|\Psi'_1\rangle_{Z \cdots CBA}$ has the same form as that of the state $|\varphi\rangle_{Z \cdots CBA}$ and can be used as the resource in next round of concentration. By iterating the entanglement concentration process n times, the total success probability that the N parties

obtain a system in a standard N -photon W state from each system in a partially entangled N -photon W -class state is

$$\begin{aligned}
P'(n) &= N \left[\frac{\alpha_1^2 \beta_1^2}{\alpha_1^2 + \beta_1^2} + \frac{\alpha_1^4 \beta_1^4}{(\alpha_1^2 + \beta_1^2)(\alpha_1^4 + \beta_1^4)} \right. \\
&+ \frac{\alpha_1^8 \beta_1^8}{(\alpha_1^2 + \beta_1^2)(\alpha_1^4 + \beta_1^4)(\alpha_1^8 + \beta_1^8)} + \cdots \\
&\left. + \frac{\alpha_1^{2n} \beta_1^{2n}}{(\alpha_1^2 + \beta_1^2)(\alpha_1^4 + \beta_1^4) \cdots (\alpha_1^{2n} + \beta_1^{2n})} \right]. \quad (26)
\end{aligned}$$

IV. DISCUSS AND SUMMARY

By far, there are no ECP for photon systems in a known W -class state, although there are two ECPs for photon system in an unknown W -class state [59, 60]. In fact, in a practical application of entanglement concentration, it is not difficult for the N parties in quantum communication to obtain information about the W -class state shared by them. They need only measure a subset of samples. The present ECP is the first one for a known W -class state and it is more practical in the application in future. Compared with other two ECPs for W -class states [59, 60], the present ECPs has some advantages. First, the present ECP requires only an N -photon system in each round of entanglement concentration, not two copies of two N -photon entangled systems, which decreases the difficulty of its implementation largely. Second, only one of the N parties in quantum communication perform the local unitary operation for reconstructing the standard W state from the W -class state and she need only communicate the classical information to other parties for retaining or discarding their photons, which greatly simplifies the complication of classical communication, similar to the works for a Bell-type state [54, 55]. Third, it has a higher success probability than others as its total success probability approaches to the limit in theory. These advantages maybe makes our ECP more feasible than other ECPs.

In summary, we have proposed an ECP for nonlocal N -photon systems in a partially entangled pure W -class state, resorting to ancillary single photons and parity-check measurement based on cross-Kerr nonlinearity. Only one of the N parties in quantum communication prepares ancillary photons and operates the entanglement concentration process for obtaining the standard N -photon W state from each partially entangled pure W -class state. She need only tell other parties to retain or discard their photons in the whole entanglement concentration, which greatly simplifies the complication of classical communication, similar to the works for a Bell-type state [54, 55]. Third, it has a higher total success probability approaching to the limit in theory by iterating the entanglement concentration process several times. All these advantages make our ECP more feasible than others. It maybe have good applications in quantum communication in future.

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